

# SQUEEZED LIGHT FROM MULTI-LEVEL CLOSED-CYCLING ATOMIC SYSTEMS

Min Xiao

*Department of Physics, University of Arkansas  
Fayetteville, Arkansas 72701*

Yi-fu Zhu

*Schlumberger-Doll Research  
Ridgefield, CT 06877*

## Abstract

Amplitude squeezing is calculated for multi-level closed-cycling atomic systems. These systems can lase without atomic population inversion in any atomic bases. Maximum squeezing is obtained for the parameters in the region of lasing without inversion. A practical four-level system and an ideal three-level system are presented. The latter system is analysed in some detail and the mechanism of generating amplitude squeezing is discussed.

## 1 INTRODUCTION

Generation of squeezed states of light has attracted lots of attention in the past decade due to the possible applications in various fields of physics. Squeezed light was generated in several different systems (atomic multi-wave mixing, degenerate parametric oscillator, optical fibers, optical diodes, second harmonic generation, and et al) and in several different forms (c.w. and pulsed) in laboratories around the world.[1] Although, this kind of quantum state is generated routinely in the laboratories, to make a compact, efficient, reliable, and cw squeezed coherent light source is still a challenge. Diode laser is a very good device to generate amplitude squeezed light with high efficiency[2], but the wavelength selection is very limited for applications in atomic spectroscopy.

Another area of recent interest in physics is to achieve lasing without atomic population inversion in multi-level atomic systems.[3-7] The possible applications of these new lasers include reaching new wavelengths and getting "quieter" laser output intensity. Several theoretical works were published in studying quantum statistical properties of  $\Lambda$ -type three-level atomic system and multi-level Raman systems.[8-11] In this paper, we discuss quantum statistical properties of multi-level closed-cycling atomic systems, which exhibits lasing without atomic population inversion.

We have studied two particular atomic systems. The first one is a practical four-level closed-cycling atomic system.[11] A brief discussion of the steady-state behaviors and linearized fluctuations in this system are given. Conditions achieving good squeezing are described.

To understand the mechanism of generating squeezing in this relatively complicated system, we constructed a simple, idealized three-level model which eliminates the intermediate level  $|4\rangle$  and neglects decay from the upper lasing level  $|2\rangle$  to the ground state  $|1\rangle$ . After adiabatic elimination of extra atomic variables (under condition  $\gamma_{32} \gg \kappa$ , with  $\kappa$  as cavity decay rate), we obtain a set of equations which are similar to an effective two-level atomic system. Then steady-state correlation functions of the amplitude fluctuations are calculated, analytically, to find best conditions for maximum squeezing. Some interesting effects are discussed and compared to the results of previous four-level model

## 2 Four-Level Cycling System

Our model consists of an ensemble of  $N$  closed four-level atoms confined in a single-mode cavity with photon loss rate  $2\kappa$ . The transition  $|1\rangle \leftrightarrow |3\rangle$  of frequency  $\omega_{31}$  is driven by a laser of frequency  $\omega_1$  with Rabi frequency  $2\Omega$ .  $2\gamma_{ij}$  ( $i, j=1-4$ ) are the spontaneous decay rates from state  $|i\rangle$  to state  $|j\rangle$ . Using standard procedure, a set of stochastic differential equations are derived. The steady state behaviour and conditions to achieve lasing without inversion were discussed in our earlier publication.[11] When  $\gamma_{21}/\gamma_{34} < 1$ , lasing will start from population inversion. As the laser intensity building up, the population of the upper lasing level  $|2\rangle$  will be depleted and the lasing will be sustained by the coherence induced between levels  $|2\rangle$  and  $|3\rangle$ . This transition from lasing with inversion to lasing without inversion in the same system is an interesting phenomena to study. At the opposite limit, i.e.  $\gamma_{21}/\gamma_{34} > 1$ , the laser will always operate with no population inversion.

We calculated, numerically, the amplitude fluctuations of the system by linearization around steady-state solutions and found that large degree of squeezing (about 80%) at the laser output can be obtained with relative low pumping power and very small decay rate from the upper lasing state  $|2\rangle$  to the lower lasing state  $|1\rangle$ .

## 3 Three-Level Idealized Cycling System

Due to the complication in the four-level system, numerical calculation has to be used in calculating the laser intensity fluctuations. In order to understand the mechanism and the limiting conditions for achieving optimal squeezing, we simplify the four-level model to be an ideal three-level closed-cycling system.

Since we are only interested in the optimal conditions for generating squeezing, the decay from upper lasing level  $|2\rangle$  to the lower lasing level  $|1\rangle$  is neglected. To simplify our calculation, we also eliminate the intermediate state  $|4\rangle$  and neglect decay from level  $|3\rangle$  to level  $|1\rangle$ . Since the effective coupling between the atomic transition and the intracavity field is  $\sqrt{N}g$  instead of  $g$ , we can increase the coupling by putting large number of atoms in the cavity mode, which is usually true for a laser system. This will justify the approximation of neglecting decay rates but of keeping finite dipole coupling between level  $|3\rangle$  and level  $|1\rangle$  and between level  $|2\rangle$  and level  $|1\rangle$ . Careful choice of the atomic element as the gain medium can also help to satisfy this approximation. The decay rate  $\gamma$  from level  $|3\rangle$  to level  $|2\rangle$  is the only one to keep.

Using the same standard procedure, we derive a set of stochastic differential equations for this system from Hamiltonian, as following:

$$\begin{aligned}
\dot{J}_{12} &= i\Omega J_{23}^\dagger + i\alpha g(J_{22} - J_{11}) + \Gamma_3(t), \\
\dot{J}_{12}^\dagger &= -i\Omega J_{23} - i\alpha^\dagger g(J_{22} - J_{11}) + \Gamma_{10}(t), \\
\dot{J}_{13} &= -\gamma J_{13} + i\Omega(J_{33} - J_{11}) + i\alpha g J_{23} + \Gamma_4(t), \\
\dot{J}_{13}^\dagger &= -\gamma J_{13}^\dagger - i\Omega(J_{33} - J_{11}) - i\alpha^\dagger g J_{23}^\dagger + \Gamma_9(t), \\
\dot{J}_{23} &= -\gamma J_{23} + i\Omega J_{12}^\dagger + i\alpha^\dagger g J_{13} + \Gamma_5(t), \\
\dot{J}_{23}^\dagger &= -\gamma J_{23}^\dagger - i\Omega J_{12} - i\alpha g J_{13}^\dagger + \Gamma_8(t), \\
\dot{J}_{22} &= 2\gamma J_{33} - i\alpha g J_{12}^\dagger + i\alpha^\dagger g J_{12} + \Gamma_6(t), \\
\dot{J}_{33} &= -2\gamma J_{33} + i\Omega J_{13} - i\Omega J_{13}^\dagger + \Gamma_7(t), \\
\dot{\alpha} &= -\kappa\alpha - igJ_{12} + \Gamma_1(t), \\
\dot{\alpha}^\dagger &= -\kappa\alpha^\dagger + igJ_{12}^\dagger + \Gamma_2(t),
\end{aligned} \tag{1}$$

where  $\langle \Gamma_i(t)\Gamma_j(t') \rangle \equiv D_{ij}\delta(t-t')$  describing the correlation of the fluctuations. There are 36 nonzero  $D_{ij}$  terms for this particular system. To save space, these nonzero diffusion terms are not given explicitly here. For a closed system, we have condition

$$J_{11} + J_{22} + J_{33} = N. \tag{2}$$

These ten differential equations are still too complicated to calculate correlations for the fluctuations analytically. However, in good cavity limit, i.e.  $\gamma \gg \kappa$  (which is a good approximation in a realistic laser system), some of the atomic variables ( $J_{13}$ ,  $J_{13}^\dagger$ ,  $J_{23}$ ,  $J_{23}^\dagger$ ,  $J_{22}$ , and  $J_{33}$ ) can be adiabatically eliminated from equation (1). We can do it by letting time derivatives over these atomic variables go to zero, because they decay much faster to their steady-state values comparing to  $J_{12}$ ,  $J_{12}^\dagger$ , and the field variables. We can then solve  $J_{13}$ ,  $J_{13}^\dagger$ ,  $J_{23}$ ,  $J_{23}^\dagger$ ,  $J_{22}$ , and  $J_{33}$  together with their corresponding fluctuation terms from equation (1) and substitute them into equations for  $J_{12}$ ,  $J_{12}^\dagger$ ,  $\alpha$ , and  $\alpha^\dagger$ . After some algebra, we arrive at

$$\begin{aligned}
\frac{\partial J_{12}}{\partial \tau} &= ixN - \left[ \frac{Y(X+2)}{2(X+1)} + \frac{X(X+1)}{Y} \right] J_{12} + x^2 \left[ \frac{Y}{2(X+1)} + \frac{X+1}{Y} \right] J_{12}^\dagger \\
&\quad + \Gamma_{12}(t), \\
\frac{\partial J_{12}^\dagger}{\partial \tau} &= -ix^\dagger N + x^{*2} \left[ \frac{Y}{2(X+1)} + \frac{X+1}{Y} \right] J_{12} - \left[ \frac{Y(X+2)}{2(X+1)} + \frac{X(X+1)}{Y} \right] J_{12}^\dagger \\
&\quad + \Gamma_{12}^\dagger(t), \\
\frac{\partial x}{\partial \tau} &= -i\frac{1}{n_0} J_{12} - \kappa x,
\end{aligned} \tag{3}$$

$$\frac{\partial x^\dagger}{\partial \tau} = i \frac{1}{n_0} J_{12}^\dagger - \kappa x^\dagger.$$

Where the new normalized variables and parameters are defined as

$$X = x^* x \equiv \frac{\alpha^\dagger \alpha}{n_0}, \quad Y \equiv \left( \frac{\Omega}{\gamma} \right)^2, \quad \tau \equiv \gamma t, \\ n_0 \equiv \frac{\gamma^2}{g^2}, \quad k \equiv \frac{\kappa}{\gamma}. \quad (4)$$

The new fluctuation terms are still quite complicated.

The steady-state equation can be easily solved to give

$$X_0 = \frac{1}{2} \left[ -1 + \sqrt{2YG + 1 - 2Y^2} \right], \quad (5)$$

where  $G \equiv Ng^2/\kappa\gamma$  is the normalized coupling strength between the field and atoms. In this calculation, we have assumed the resonance condition between the cavity mode and the transition frequency from level  $|2\rangle$  to level  $|1\rangle$ . The normalized steady-state population distributions are

$$\frac{J_{11}}{N} = \frac{1}{2} - \frac{Y}{2G}, \\ \frac{J_{22}}{N} = \frac{1}{2} + \frac{Y}{2G} - \frac{1}{2G} \left( -1 + \sqrt{2YG + 1 - 2Y^2} \right), \\ \frac{J_{33}}{N} = \frac{1}{2G} \left( -1 + \sqrt{2YG + 1 - 2Y^2} \right). \quad (6)$$

To calculate correlation fuctions of the fluctuations, we need to linearize equation (3) around their steady-state solutions. From the numerical calculation of the four-level system and the steady-state solutions of this system, we can determine the area in the parameter space where best squeezing occurs. For  $X_0 \gg 1$ , and small, but finite,  $Y$ , equations in (3) can be linearized to give

$$\frac{d}{d\tau} \begin{pmatrix} \delta x \\ \delta x^\dagger \\ \delta J_{12} \\ \delta J_{12}^\dagger \end{pmatrix} = - \begin{pmatrix} k & 0 & i\frac{1}{n_0} & 0 \\ 0 & k & 0 & -i\frac{1}{n_0} \\ -iA & iB & C & -C \\ -iB & iA & -C & C \end{pmatrix} \begin{pmatrix} \delta x \\ \delta x^\dagger \\ \delta J_{12} \\ \delta J_{12}^\dagger \end{pmatrix} \\ + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & D & -D \\ 0 & 0 & -D & D \end{pmatrix}^{\frac{1}{2}} \begin{pmatrix} \xi_1(\tau) \\ \xi_2(\tau) \\ \xi_3(\tau) \\ \xi_4(\tau) \end{pmatrix}, \quad (7)$$

where

$$A \equiv kn_0 \left( G - \frac{5X_0^2}{Y} - \frac{Y}{2} \right),$$

$$B \equiv kn_o(\frac{3X_o^2}{Y} - \frac{Y}{2}),$$

$$C \equiv \frac{X_o^2}{Y} + \frac{Y}{2}, \quad (8)$$

$$D \equiv kn_o(-20\frac{X_o^3}{Y} + 16X_o^2 - X_oY - Y^2 + \frac{4X_o^4}{Y^2}).$$

The intracavity correlation functions and, therefore the amplitude squeezing is easily calculated by standard method. The intracavity squeezing is given by

$$S_+ = 2n_o(< \delta x \delta x^\dagger > + < \delta x \delta x >)$$

$$\approx \frac{-20\frac{X_o^3}{Y} + 16X_o^2 - X_oY - Y^2 + \frac{4X_o^4}{Y^2}}{(\frac{X_o^2}{Y} + \frac{Y}{2})(G - \frac{10X_o^2}{Y} - Y)}. \quad (9)$$

The steady-state intensity  $X_o$  is related to the pumping intensity  $Y$  and coupling strength  $G$  through the steady-state equation (5).

It is easy to show that  $S_+$  is limited to the minimum value of -0.5, which corresponding to a 50% squeezing inside cavity. The output spectrum of squeezing has also been calculated numerically. The maximum squeezing is also limited to a 50% level at around carrier frequency.

## 4 Discussion

We have calculated two atomic models for generating squeezed states of light and conditions for lasing without inversion. The four-level system can start to lase with or without population inversion depending on the relative decay rates. Far above threshold the lasing is only sustained by the coherence induced between level  $|3\rangle$  and level  $|2\rangle$ . For the three level ideal model, the laser will always operate without the population inversion.

The four-level system can generate 80% squeezing at the output for relatively low pumping power. As for the simplified and idealized three-level cycling system, even without decay from the upper lasing state to the lower lasing state, the maximum output squeezing and the total intracavity squeezing are both limited to the 50% level. we realize that 50% squeezing comes from the suppression of spontaneous emission of the upper lasing level. The extra 30% squeezing is due to the pumping regulation through the incoherent decay processes between level  $|3\rangle$  & level  $|4\rangle$  and between level  $|4\rangle$  & level  $|2\rangle$ . So, the extra level  $|4\rangle$  actually enhances the available squeezing for the laser output.

### References

- [1] See, e.g., the feature issue on Squeezed States of the Electromagnetic Field, J. Opt. Soc. Am. B4, 1450-1741, (1987); and Quantum Fluctuations in Optical Systems, Progress in Optics, XXX, 1-85 (1992).
- [2] Y. Yamamoto, S. Machida, and Y. Itaya, Phys. Rev. Lett. 58, 1000 (1987).
- [3] S.E. Harris, Phys. Rev. Lett. 62, 1033 (1989); A. Imamoglu, Phys. Rev. A40, 4135 (1989).

- [4] M.O. Scully, S.Y. Zhu, and A. Gavrielides, Phys. Rev. Lett. 62, 2813 (1989); E.F. Fill, M.O. Scully, and S.Y. Zhu, Opt. Commun. 77, 36 (1990).
- [5] A. Nottelmann, C. Peters, and W. Lange, Phys. Rev. Lett. 70, 1783 (1993).
- [6] E.S. Fry, et al, Phys. Rev. Lett. 70, 3235 (1993).
- [7] W.E. van der Veer, et al, Phys.Rev.Lett.70,3243 (1993).
- [8] G.S. Agarwal, Phys. Rev. Lett. 67, 980 (1991).
- [9] K.M. Gheri and D.F. Walls, Phys. Rev. Lett. 68, 3428 (1992); Phys. Rev. A45, 6675 (1992).
- [10] H. Ritsch, M.A.M. Marte, and P. Zoller, Europhys. Lett. 19, 7 (1992); H. Ritsch, M.A. Marte, Phys. Rev. A47, 2354 (1993).
- [11] Yifu Zhu and Min Xiao, Phys. Rev. A48, 3895 (1993).